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7. Additional tasks for exercise on „Introduction to Numerical Mathematics“

Problem 28:

(a) For $k \in \mathbb{N}$ compute the value

$$s = \lim_{n\to\infty} k\underbrace{\sqrt{\frac{3}{4}k^2 + 2k + 1 + k\sqrt{\frac{3}{4}k^2 + 2k + 1 + ... + k\sqrt{\frac{3}{4}k^2 + 2k + 1}}}}_{n \text{ times root}} .$$

Specify a step function φ and an interval I such that the iteration $x_{n+1} = \varphi(x_n)$ converges to s for all initial values $x_0 \in I$. Compute 5 iterations with a proper starting value.

What is the exact value of s for a fixed k?

(b) If you enter a number $x \in (0, \pi)$ into a calculator and press the "cos" key several times, you observe numerically a convergence to a fixed-point. Analyze this behavior with Banach's fixed-point theorem (Specify an interval, check preconditions, specify Lipschitz constant).

Problem 29:

Compute the intersection of the curves $f(x) = \ln(16 - x)$ and $g(x) = \sqrt{\frac{2}{3}x^2 + 4}$ in $[1, 7]$. To this end use a fixed-point iteration

(a) by isolating x on the left hand side of $f(x) = g(x)$,

(b) by isolating x on the right hand side of $f(x) = g(x)$.

(c) Analyze both iterations in terms of convergency.

(d) How many iterations are necessary at the latest to drop with the distance $|x_i - x^*|$ below a level of $\varepsilon = 10^{-5}$?

Problem 30:

Apply Newton's method to solve $f(x) = x^3 - 2x + 2 = 0$. Perform three steps for each start value.

(a) Use $x_0 = -1$.

(b) Use $x_0 = 1$.