

1. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 1:

Check for which values $n \leq 6$ the calculation of $S_n = \sum_{i=1}^n \frac{1}{n}$ will give the correct result $S_n = 1$. Note the floating point arithmetic $x \oplus y = \text{rd}(x + y)$, that is, floating point numbers are first added exactly and then the rounding operator is applied.

Problem 2:

Sum up at least ten billion terms of the harmonic series. Compare the results for summation in forward and backward direction and explain the difference.

Problem 3:

Calculate the value of the polynomial $p(x) = 1.0837x^4 + 2.7911x^3 + 0.75149x^2 - 5.8205x - 7.6123$ for $x = 1.4935$ using the following two algorithms:

Algorithm 1	Algorithm 2
$y_1 = x$	$y_1 = 1.0837x$
$y_2 = xy_1$	$y_2 = (y_1 + 2.7911)x$
$y_3 = xy_2$	$y_3 = (y_2 + 0.75149)x$
$y_4 = xy_3$	$y_4 = (y_3 - 5.8205)x$
$y = 1.0837y_4 + 2.7911y_3 + 0.75149y_2 - 5.8205y_1 - 7.6123$	$y = y_4 - 7.6123$

Compare both algorithms according their computation expense and memory consumption.

Problem 4:

- a) Show that the inequality

$$\frac{1}{1+2x} - \frac{1-x}{1+x} > 0$$

holds for each positive real number, $x \in \mathbb{R}$, $x > 0$. Calculate the left-hand side of the inequality for $x = 10^{-10}$ and provide a numerically stable form of the inequality.

- b) Show that the function

$$f(x) := x(\exp(x^{-1}) - 1)$$

has the limit 1 for $x \rightarrow \infty$. Use a calculator to evaluate the function $f(x)$ for $x = 10^j$ ($j = 5, \dots, 15$). Provide a numerically stable form using a series expansion.

Problem 5:

Evaluate Taylor's series of $\exp(x)$ at $x = 0$ and for different values of h and different orders. Let $T_m(h)$ be the approximating value of order $m = 1, 2, 3$. Use $h = 10^{-i}$, $i = 1, \dots, 4$ and plot the errors in a double logarithmic plot, thus one axis is h and the other is $|\exp(x+h) - T_m(h)|$ and estimate the slopes of $T_m(h)$.

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.