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2. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 6:
Compute the value

$$s = \lim_{k\to\infty} \underbrace{\frac{1}{1+\frac{1}{1+\frac{1}{1+\dots}}}}_{k \text{ fractions}}.$$

Proceed as follows for this purpose:

(a) Specify a step function φ with s being a fixed point of φ.

(b) Specify an interval $I = [\alpha, \beta]$ with $\alpha < \beta$ so that the requirements of Banach's fixed point theorem are met. Check them.

(c) Compute 3 iterations with a valid $x_0 \in I$.

(d) What is the exact value?

Problem 7:
If you enter a number $x \geq 0$ in a calculator and press the root-key ($\sqrt{\ }$) several times, you observe numerically a convergence to a fixed-point. Analyze this behavior in terms of Banach's fixed-point theorem, check the preconditions and determine Lipschitz constant.

Problem 8:
The function $f(x) = \cos^2 x$ has exactly one fixed-point. Specify a largest possible interval (a, b) such that the conditions of Banach's fixed-point theorem are satisfied for any closed $I \subset (a, b)$.

Consider once again problem 30. Let $x_0 = 0.6$. Give an a-priori estimation for the error after k steps as accurate as possible. Determine for this, starting from x_0, a Lipschitz constant L as small as possible so that a contraction is present, and use L for the estimation. After how many steps is x_k certainly not further away from the fixed-point than $\varepsilon = 10^{-2}$?

Compute three steps and an a-posteriori error estimation for the last two iterations.

Problem 9:
Use Newton's method on the example of calculating $\sqrt{2}$ as solution of $f(x) = x^2 - 2 = 0$. Use as starting values $x_0 = 2$ and $x_0 = 100$ and as stopping criterion the error of the residual norm dropping below 10^{-6}.

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.