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4. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 13:
Formulate a fixed-point iteration to determine the point of intersection (x^*, y^*) defined by the two equations

$$f(x) = 2 \cdot \exp(-x), \qquad g(x) = \sqrt{1+x}$$

Show that the assumptions of the Banach fixed-point theorem are fulfilled and thus it holds

$$x^* = \lim_{k\to\infty} x_k\,.$$

Problem 14:
Calculate 10 steps using the simplified Newton method for problem 11 with $x_0 = 2$. Evaluate f' in the first and in the sixth iteration. Compare the results with those of problem 11.

Problem 15:
Compute approximation to the two zeros of

$$f : \mathbb{R}^2 \to \mathbb{R}^2, \qquad f(x,y) = \begin{pmatrix} \exp(x) - y \\ y^2 - x - 3 \end{pmatrix}.$$

(a) Give the multidimensional iteration prescription of Newton's method for this $f(x,y)$.

(b) Apply one step of Newton's method for $x^{(0)} = (-1, 0)^T$.

(c) Further, apply also one step of Newton's method for $x^{(0)} = (1, 4)^T$.

(d) For which $(x, y) \in \mathbb{R}^2$ is $f'(x, y)$ not invertible?

(e) Use an (unfavorable) alternative and apply (compute two steps to for $x^{(0)} = (-1, 0)^T$) the method of steepest descend without stepsize control to minimize

$$F : \mathbb{R}^2 \to \mathbb{R}, \quad F(x) = \frac{1}{2}\|f(x)\|_2^2.$$

Problem 16:
Find the interpolating polynomial of smallest possible degree through the points $(0, 4)$, $(1, 7)$, $(3, 31)$ and $(2, 14)$. Use Lagrangian base-polynomials.

Add afterwards the point $(x_3, f_3) = (4, 3)$ to your interpolation polynomial!

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.