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5. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 17:
To indicate the efficiency η of a heat pump depending on the temperature T a manufacturer specifies three pairs of values as follows

T [in °C]	-7	2	10
η	2	3.5	4.5

.

(a) What efficiency can be assumed for $T = 5°\text{C}$? For this, determine the interpolation polynomial $p(x)$ for these points (use a method of your choice) and compute $p(5)$.

(b) Tests suggest that $\eta(0) = 3$. Compute the interpolation polynomial regarding this additional information using Newton's scheme.

(c) Omit the assumption from (b) and assume instead that the machine has its maximal efficiency for $T = 10°\text{C}$. Compute the interpolation polynomial taking this information into account.

Problem 18:
Rostock averaged 6.5° C degrees in March, 11.1° C in April, 20.1° C in June, and 22.0° C in July. Approximate the temperature for May.

(Units dropped, all quantities made dimensionless.)

Problem 19:
A prismatic body of length $L = 3$, width and height are equal to 1, is made of a composite material with smoothly changing density $\rho = \rho(x)$, $x \in [0, L]$. From measurements we know that $\rho(0) = 3$, $\rho(1) = \rho(3) = 1$.

Calculate the total mass of the body, using polynomial interpolation of ρ!

Problem 20:
Calculate the Newton interpolation polynomial for

$$p(0) = 1,\ p(-1) = 3,\ p(1) = 15,\ p'(-1) = -12,\ p'(1) = 40.$$

Compute $p'(0)$.

Problem 21:
For a function $u : \mathbb{R}^2 \to \mathbb{R}$ the following points are known

$$u(0,1) = 3,\ u(1,1) = 6,\ u(0,2) = 7,\ u(1,2) = 11,\ u(1.5,1.5) = 10.75.$$

Further u should satisfy $\Delta u(1.5, 1.5) = 0$.

Set up the 6 equations necessary to determine the two-dimensional polynomial of degree less than or equal to two using this information but do not finally compute the polynomial.

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.