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6. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 22:

(a) For $f(x) = \tan(x)$, determine approximations to $f'(x_0)$ for $x_0 = 0.125\pi$ using forward, backward and central difference and for $f''(x_0)$ using second order central difference. Compare the approximate values with the actual ones. Use $h = 10^{-3}$ for all approximations.

(b) Determine for $u(x, y) = \sin(x)\exp(-y^2)$ approximations to $u_{xx}(x_0, y_0)$, $u_{xy}(x_0, y_0)$ and $u_{yy}(x_0, y_0)$ for $(x_0, y_0) = (1.25, 0.75)$. Use $h_x = h_y = h$ for $h = 10^{-3}$. What are the absolute errors.

(c) Compute an approximation of the Jacobian $J_f(-1, 0)$ for f from problem 31 at the point $x = (-1, 0)$. Use forward differences and $h = 10^{-4}$. Compare the approximation with the exact value and calculate $\operatorname{cond}_\infty(A)$.

Problem 23:

When modeling the deformation of a one-dimensional beam, a difference quotient is needed for approximation $u^{(4)}(x) = \frac{\mathrm{d}^4 u}{\mathrm{d}x^4}(x)$. Derive this using Taylor expansions of $u(x \pm h)$ and $u(x \pm 2h)$ and $u(x)$. What is the error order of the approximation.

Use the difference quotient to approximate $u^{(4)}(0.25)$ for $u(x) = -\cos(\pi x)$ with $h = 10^{-k}$, $k = 1, \dots, 4$. Give the absolute errors.

Problem 24:

For $f(x) = \sin(\pi x)$ use the composite trapezion rule as well as the composite Simpson's rule to approximate

$$I = \int_0^1 f(x)\,\mathrm{d}x.$$

(a) Apply both rules for $n = 6$ and give the errors $|I - T_6|$ and $|I - S_6|$.

(b) Determine n such that the error of the composite trapezion rule resp. the composite Simpson's rule is smaller than 10^{-6}.

Problem 25:

Compute appoximations to

$$\int_0^1 \sin(\pi x)\,\mathrm{d}x$$

(a) using Gaussian quadratur rules with 2 nodes,

(b) using Gaussian quadratur rules with 3 nodes,

(c) using composite Gaussian quadratur rules with 2 nodes and 3 intervals.

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.