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# 7. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 26:
The height of a barrel is 0.6 m and its diameter follows the function $d(h) = 0.3 - 0.4(h-0.6)^2$. Determine the volume of this barrel. To this end calculate the volume of the solid of revolution that results when the function $d(h)$ rotates around the $h$-axis. Use composite Simpson's rule with $n = 4$ intervals.

Problem 27:
Consider the function

$$f : \mathbb{R}_+ \to \mathbb{R}, \quad f(x) = \frac{1}{x^2}.$$

(a) Use composite Simpson's rule $S_4$ with $n = 4$ subintervals to approximate $I = \int_1^5 f(x)\,\mathrm{d}x$.

(b) How large must $n$ be in order to be sure that the error is smaller than $\varepsilon = 10^{-4}$?
HINT: For Simpson's rule holds

$$\left| \int_a^b f(x)\,\mathrm{d}x - S_n \right| \leq \frac{(b-a)h^4}{180} \max_{x\in[a,b]} |f^{(4)}(x)|.$$

(c) Approximate $\int_1^\infty f(x)\,\mathrm{d}x$ and use the coordinate transformation $x(z) = \tan(z)$ with

$$x' = \frac{\mathrm{d}x}{\mathrm{d}z} = \frac{1}{\cos(z)^2} \quad \Rightarrow \quad \mathrm{d}x = \frac{1}{\cos(z)^2}\,\mathrm{d}z$$

and composite Gaussian rule $Q_1$ with 2 nodes and 2 intervals.

Problem 28:
Compute an approximation to $\int_{-\infty}^{\infty} \exp(-x^2)\,\mathrm{d}x$. To this end use the coordinate transformation $x(z) = \tan(z)$ with

$$x' = \frac{\mathrm{d}x}{\mathrm{d}z} = \frac{1}{\cos(z)^2} \quad \Rightarrow \quad \mathrm{d}x = \frac{1}{\cos(x)^2}\,\mathrm{d}z$$

and Simpson's rule for $n = 6$. Give the errors.

Problem 29:
Consider the initial value problem

$$y'(x) = -\sin(x)y(x) \text{ for } x \in [0,1], \quad y(0) = 1.$$

(a) Compute three steps of Euler's method with $h = 0.1$.

(b) Compute two steps of Heun's method with $h = 0.15$.

(c) Apply the 4-stage Runge-Kutta method RK-4 and compute one step with $h = 0.3$.

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The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.