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8. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 30:

```
n = 51;
h = 10/(n-1);
X = 0:h:10;
Y(1) = exp(cos(0));
for i=1:n-1
    K1 = -sin(X(i))*Y(i);
    K2 = -sin(X(i)+h/2)*(Y(i)+h/2*K1);
    K3 = -sin(X(i)+h/2)*(Y(i)+h/2*K2);
    K4 = -sin(X(i)+h)*(Y(i)+h*K3);
    Y(i+1) = Y(i)+h/6*(K1+2*K2+2*K3+K4);
end
```

Refer to the relevant line(s) for each of the following questions.

(a) Which initial value problem is solved here?

(b) What is the numerical method used and what is the step size?

(c) For the selected step size, the maximum error is $7.1 \cdot 10^{-6}$. Which error is to be expected due to the consistency order of the method, if $n = 101$ would be chosen?

Problem 31:
Consider the initial value problem $y'(x) = -5y(x)$ for $x \in [0, 5]$ with $y(0) = 1$.

(a) Apply two steps of Euler's method using $h = 0.5$.

(b) Apply one step of the classical Runge-Kutta method using $h = 1$.

(c) For this ODE and Euler's method with $h = 0.5$ it is possible to derive an explicit formula for Y_{k+1} depending only on k. Determine this formula and compute Y_{10} without computing $Y_2, \ldots, Y_9$.

(d) Show that the application of the method $Y_{k+1} = Y_k + hf\big(x_k + h/2, (Y_k + Y_{k+1})/2\big)$ to the problem results in

$$Y_n = Y_0 \cdot \left(\frac{1 - 2.5h}{1 + 2.5h}\right)^n.$$

Compute Y_{10} for $h = 0.5$.

Problem 32:
Consider Crank-Nicolson's method

$$Y_{k+1} = Y_k + \frac{h}{2}\big(f(x_k, Y_k) + f(x_{k+1}, Y_{k+1})\big).$$

(a) Show that the method has at least order 2.

Hint: To this end use $f(x + h, y(x + h)) = f(x + h, y(x) + hy'(x) + \mathcal{O}(h^2))$.

(b) Determine the stability domain of the method (model probelm $y' = \lambda y$).

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.