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Institute of Mathematics
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10. Tutorial on the lecture „Introduction to Numerical Mathematics"

Problem 35:
Let $u(x)$ be the vertical deformation of a beam that is fixed at both ends by the force $f(x)$. The associated boundary value problem for $x \in (0,1)$ is

$$u^{(4)}(x) = f(x), \qquad u(0) = u(1) = u'(0) = u'(1) = 0.$$

An equidistant discretization with $x_0, \ldots, x_{N+1}$ and the finite difference

$$u^{(4)}(x_i) \approx h^{-4}(U_{i-2} - 4U_{i-1} + 6U_i - 4U_{i+1} + U_{i+2}) \text{ for } 2 \le i \le N-1$$

results in a system of linear equations for the unknown $U_i \approx u(x_i)$. The boundary conditions result in U_0, U_1, U_N and U_{N+1}. Give the system of linear equations for the calculation of $U_2, \ldots, U_{N-1}$ for $N = 20$. Apply the external force

$$f(x) = \begin{cases} -1 & \text{for } x \in [0.6, 0.8] \\ 0 & \text{otherwise} \end{cases}$$

and solve the system by Jacobi's method. Use $x^{(0)} = b$ as the initial guess and $\|Ax^{(i)} - b\| \le 10^{-6})$ as the stopping criterion. Plot the solution.

Problem 36:
Consider the system of linear equations $Ax = b$ with

$$A = \begin{pmatrix} \alpha & 1 & 0 \\ 1 & 0 & 1 \\ 2 & \alpha & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$$

for a parameter $\alpha \in \mathbb{R}$.
  (a) Calculate the solution x depending on the parameter α.
  (b) For which values α does no solution exists?
  (c) Which values α cause a Gaussian elimination that requires pivoting?
  (d) Determine a right-hand-side $b \ne (0,0,0)^T$ such that there exists a solution for any choice of the parameter α.

Problem 37:

Consider an electric circle with 5 resistors placed on the edges of the square $ABCD$ and on its diagonal BD. Resistor $R_1 = 3\Omega$ connects A with B, resistor $R_2 = 8\Omega$ connects B with C, resistor $R_3 = 3\Omega$ connects C with D, resistor $R_4 = 8\Omega$ connects D with A and resistor $R_5 = 3\Omega$ connects B with D. A current of 1A enters the electric circle at A and leaves it at C.

Use Kirchhoff's laws to derive a system of linear equations to calculate the five currents passsing the resistors and compute the solution using the program from task 8.

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.