University of Rostock                                             Rostock, 12.01.2026
Institute of Mathematics
PD Dr. M. Sawall

11. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 38:
Determine the $LU$-decomposition of the following matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \\ 1 & 16 & 81 & 256 \end{pmatrix}$$

Solve the linear system $Ax = b$ for $b = (3, 1, -15, -107)^T$ as well as for $b = (10, 20, 46, 116)^T$.

Problem 39:
Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}.$$

Calculate $\text{cond}_1(A)$, $\text{cond}_2(A)$ and $\text{cond}_\infty(A)$.

Problem 40:
Which value of $c$ leads to zero in the second pivot position of the matrix:

$$A = \begin{pmatrix} 1 & c & 0 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{pmatrix}$$

For exactly that value of $c$, calculate a $PLU$ decomposition such that $PA = LU$!

Problem 41:
Consider a band-matrix with with $\alpha$ lower subdiagonals and $\beta$ upper subdiagonals for $\alpha, \beta \in \mathbb{N}$. The $LU$-decomposition return two band-matrices. How many subdiagonals do not vanish if the Gaussian-elimination is done

(a) without pivot search and

(b) with pivot search?

Problem 42:
Are the following symmetric matrices positive/negative definite?

$$H_1 = \begin{pmatrix} 4 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}, \qquad H_2 = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 17 \end{pmatrix}.$$

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.