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Rostock, 19.01.2026

# 12. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 43:

(a) Apply Jacobi's method and Gauss-Seidel's method to solve $Ax = b$ with

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Use $x^{(0)} = (1, 1, 1)^T$ as starting vector and compute 2 steps for each method.

(b) Does the iteration converge for each starting vector?

(c) Apply an a-priori estimation of the error in order to determine how many steps are necessary to reach an approximation with $\|x^* - x^{(k)}\|_2 < 10^{-4}$.

(d) Apply Jacobi's method to solve $Ax = b$ with

$$A = \text{tridiag}(1, -2, 1) \in \mathbb{R}^{n \times n}, \quad b = (1, \ldots, 1)^T \in \mathbb{R}^n$$

for $n = 101$. Use $\|Ax^{(i)} - b\|_2 \le 10^{-6}$ as stopping criterion and $x^{(0)} = -b$ as initial guess. How many iterations are necessary for the Gauss-Seidel iteration?

Problem 44:

Compute approximations to the solution of the system of linear equations $Ax = b$ with

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 1 \\ 1 & 0 & 2 \end{pmatrix}, \qquad b = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}.$$

(a) Calculate one step using Gauss-Seidel's method starting with $x^{(0)} = (1, 1, 1)^T$.

(b) Calculate one step using Jacobi's method starting with the same $x^{(0)}$.

(c) Compute the error propagation matrix of Jacobi's method for this certain problem.

Problem 45:

Regard the system of linear equations $Ax = b$ with

$$A = \begin{pmatrix} 1 & 0 & \gamma \\ \alpha & 1 & 0 \\ 0 & \beta & 1 \end{pmatrix} \quad \text{mit} \quad \alpha, \beta, \gamma \in \mathbb{R}, \quad b \in \mathbb{R}^3.$$

Under which conditions at $\alpha$, $\beta$ and $\gamma$ does the Gauss-Seidel method converge for this $A$ and all starting vectors? Determine the error propagation matrices $F_J$ and $F_{GS}$ and their characteristic polynomials $p_J(\lambda)$ and $p_{GS}(\lambda)$. Calculate the zeros of $p_J(\lambda)$ and estimate the magnitudes of the zeros of $p_{GS}(\lambda)$ and use a theorem from the lecture.

Show that for this $A$, the Gauss-Seidel method converges for all starting vectors if and only if Jacobi's method converges for all starting vectors.

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The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.