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13. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 46:
Using Gershgorin's circle theorem, calculate intervals for the eigenvalues of

$$A = \begin{pmatrix} 6 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & -4 \end{pmatrix}, \qquad B = 16 \begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & -1 \\ 0.1 & 2 \end{pmatrix}.$$

So which of the matrices are safely invertible without further calculations? Estimate for these the eigenvalues of the inverses.

Problem 47:
Apply the power method as well as the inverse power method to A from problem 46. Calculate three steps starting from $x^{(0)} = \sqrt{1/3} \cdot (1,1,1)^T$ for the power method as well as one step starting from $z^{(0)} = \sqrt{1/2} \cdot (1,-1,0)^T$ for the inverse vector iteration. Give the approximations for the largest and smallest eigenvalues of A as well as the eigenvectors.

Problem 48:
Apply 3 steps of the shifted inverse power method for $\mu = -4$ to A from task 47. Use the starting vector $x^{(0)} = \sqrt{1/3} \cdot (1,1,1)^T$ and the LU-decomposition of $A - \mu I$ with

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \qquad U = \begin{pmatrix} 2 & 1 & 2 \\ 0 & -1.5 & 0 \\ 0 & 0 & -10 \end{pmatrix}.$$

Problem 49:
Consider the symmetric matrix

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 4 \end{pmatrix}.$$

(a) Use Gerschgorin-circles to determine intervals for the eigenvalues.

(b) Apply the power method using the starting vector $x = (0,1,-2,3)^T$ without internal normalization. Compute two steps and calculate the Rayleigh-coefficient for the last iteration.

(c) Apply the inverse power method with shift for the starting vector $x = (-2,1,2,1)^T$ and the shift $\mu = 2$. Compute one step without internal normalization and the Rayleigh-coefficient.

HINT:

$$(A - 2I)^{-1} = \begin{pmatrix} -2 & 1 & 2 & 1 \\ 1 & -1 & -2 & -1 \\ 2 & -2 & -2 & -1 \\ 1 & -1 & -1 & 0 \end{pmatrix}.$$

Problem 50:
Consider a system of n equal masses with $m = M$ in a row. Each mass is connected to its left and right neighbors by equal springs. The first and last masses are connected to the left and right walls.

Let $z \in \mathbb{R}^n$ be the vector of deflections from the equilibrium positions. Find the equations of motion describing the mechanical system and state them in the form $m\ddot{z} = Az$.

Problem 51:
Consider the matrix $A = h^{-2}\text{tridiag}(-1, 2, -1) \in \mathbb{R}^{n\times n}$ for $n \in \mathbb{N}$ and $h = 1/(n+1)$. Show that the columns of the matrix $X \in \mathbb{R}^{n\times n}$ with the elements

$$X_{jk} = \sin(k\pi jh), \quad j, k \in \{1, \dots, n\},$$

are the eigenvectors of A and compute the eigenvalues.

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.