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1. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 1.1:
Find all solutions of the following ordinary differential equations/systems:

a) $y'(t) = \frac{2}{t}y(t) + 4t^3$

b) $y'(t) = \dfrac{1 + (y(t))^2}{t}$

c) $x'(t) = y(t) + t, \quad y'(t) = x(t) - 1$

Problem 1.2:
Find all solutions $u = u(x, y)$ of the partial differential equation

$$12u_y - \frac{1}{2}u_{xy} = 0.$$

Problem 1.3:
Let $z = z(x, y)$ be an unknown function. Solve the following initial value problem:

$$2xz_x + yz_y = 0, \quad z(1, y) = y^2 + 5$$

Problem 1.4:
Let ϕ be twice differentiable and differentiable for all real x. Further be $c \in \mathbb{R}$.

Show that

$$u(x,t) = \frac{1}{2}(\phi(x + ct) + \phi(x - ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} \phi(s)ds$$

is a solution of $u_{tt} = c^2 u_{xx}$.

Next show that this solution also satisfies the conditions $u(x, 0) = \phi(x); u_t(x, 0) = \phi(x)$ for all $x \in \mathbb{R}$.