

2. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 2.1:

Let f and g be arbitrary differentiable functions. Find the second order partial differential equation which is independent of f and g and has the general solution

- (a) $u(x, y) = f(x) + g(y)$,
- (b) $u(x, y) = f(x) \cdot g(y)$.

Problem 2.2:

Consider the 2D vector field $\vec{v} = ((x+1)y, x(y+1))^T$.

- (a) Compute the integral curves of \vec{v} .
- (b) Determine the general solution $z(x, y)$ of $(x+1)yz_x + x(y+1)z_y = 0$.

Problem 2.3:

Solve the PDE

$$3yu_x - 2xu_y = 0$$

and find the particular solutions that satisfy the initial condition

- (a) $u(x, y) = x^2$ on the line $y = x$ resp.
- (b) $u(x, y) = 1 - x^2$ on the line $y = -x$.

Problem 2.4:

Compute the integral curves of the following vector fields and find the curves that pass through the point $P = (1, 1, 1)$

$$(a) \vec{v} = \begin{pmatrix} z \\ y \\ x \end{pmatrix}, \quad (b) \vec{v} = \begin{pmatrix} x \\ -y \\ y^2(1-z) \end{pmatrix}, \quad (c) \vec{v} = \begin{pmatrix} y+z \\ z+x \\ x+y \end{pmatrix}.$$