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# 3. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 3.1:

(a) Find the integral surfaces of $\overrightarrow{v} = (x^2,\ y^2,\ (x+y)z)^T$.

(b) Calculate the surface containing

$$\text{(i)}\ \ x = y,\ z = 1, \qquad \text{(ii)}\ \ x = y = z, \qquad \text{(iii)}\ \ x = 1,\ y = z.$$

Problem 3.2:

Calculate the integral curves with either the integral surfaces **or** using a system of ODEs to get the parametric form.

$$\text{(a)}\ \ \overrightarrow{v} = \begin{pmatrix} x \\ x+y \\ x+y+z \end{pmatrix}, \qquad \text{(b)}\ \ \overrightarrow{v} = \begin{pmatrix} x^2-1 \\ (y^2+1)(x+1) \\ xz+x-z-1 \end{pmatrix}.$$

Problem 3.3:

(a) Find the integral surfaces of the vector field

$$\overrightarrow{v} = \begin{pmatrix} (x+1)y \\ y^2+1 \\ y(z-1) \end{pmatrix}.$$

(b) Compute the solution $z = z(x,y)$ of the inhomogeneous PDE

$$(x+1)yz_x + (y^2+1)z_y = y(z-1).$$

(c) Further, solve the PDE

$$(x+1)yu_x + (y^2+1)u_y + y(z-1)u_z = 0$$

for $u = u(x,y,z)$.

Problem 3.4:

Find the solution $z = z(x,y)$ for the PDE

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = -1, \qquad 0 < y < x$$

with the initial condition $z(x,0) = \ln x$ for $x > 0$.