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4. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 4.1:

(a) Find the integral curves of $\overrightarrow{v} = (x, y, x + y + z)^T$

(b) Calculate the general solution of $xu_x + yu_y + (x + y + z)u_z = 0$, with $u = u(x, y, z)$.

(c) Solve $xz_x + yz_y = x + y + z$ with $z = z(x, y)$ unknown.

(d) Given is the initial value problem $xz_x + yz_y = x + y + z$, $z(x, x) = g(x)$. Under which conditions at $g(x)$ is it solvable in a neighborhood of (x_0, x_0)? Find all solutions.

Problem 4.2:
Classify the following PDEs

(a) $4u_{xx} + 4u_{xy} + u_{yy} = -4u$,

(b) $x^2u_{xx} + 2u_{xy} + y^2u_{yy} = 0$,

(c) $8u_{xx}+6u_{yy}+4u_{zz}+u_{xy}+2u_{xz}+u_{yz} = 0$,

(d) $u_{xx}(x, y) + cu_{xy}(x, y) + u_{yy}(x, y) = 0$, with $c \in \mathbb{R}$,

(e) $2u_{xy} - 2u_{xz} + 2u_{yz} + 3u_x - u = 0$.

Problem 4.3:
Transform $u_{xx} + 2u_{yy} + u_{zz} - 2u_{xz} = 0$ into the canonical. To this end compute a coordinate transformation with $A \in \mathbb{R}^{3\times3}$ of the form

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = A \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

Problem 4.4:
Consider the PDE

$$21u_{xx} + 15u_{yy} + 18u_{zz} - 12u_{xz} + 12u_{yz} = 0.$$

(a) Classify the PDE.

(b) Transform it into canoncial form.

Problem 4.5:
Let $u = u(x, y)$ and $x, y \neq 0$ be given as well as the PDE

$$2x^2u_{xx} + 5xyu_{xy} + 2y^2u_{yy} + 8xu_x + 5yu_y = 0.$$

(a) Classify the PDE.

(b) Calculate the characteristic curves.

(c) Transform the PDE into canoncial form.

(d) Find the general solution of the PDE.