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5. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 5.1:

Let $u = u(x,t)$ and $k \in \mathbb{R} \setminus \{0\}$ be given together with the PDE

$$u_{xx} + 2ku_{xt} + k^2 u_{tt} = 0.$$

(a) Find the characteristic curves of the PDE.

(b) Bring the PDE into canonical form.

(c) Find the solutions of the PDE.

Problem 5.2:

(a) Show that $u(x,y) = \ln(x^2 + y^2)$ is harmonic outside the origin.

(b) Show that $u(x,y,z) = (x^2 + y^2 + z^2)^{-1/2}$ is harmonic outside the origin.

(c) If u and v and $u^2 + v^2$ are harmonic, then u and v must be constant.

Problem 5.3:

Let $\Omega = (-3,1) \times (-2,2)$ and $u : \mathbb{R}^2 \to \mathbb{R}$ be the solution of the Cauchy-problem

$$\begin{aligned} \Delta u &= 0 \quad \text{for } x \in \Omega \\ u(x) &= ax_1 - bx_2 \quad \text{for } x \in \partial\Omega \text{ and } a, b > 0 \end{aligned}$$

(a) Calculate the minimum and maximum of $u(x)$.

(b) Give the points where the minimum and maximum are reached.

Problem 5.4:

Show that for the coordinate tansformation $x = r\cos\varphi$ and $y = r\sin\varphi$ holds

(a) $\Delta u(x,y) = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\varphi\varphi}$,

(b) $u_y = u_r \frac{y}{r} + u_\varphi \frac{x}{r^2}$.