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Rostock, 15.05.2025

6. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 6.1:
Consider Ω to be a disc with Radius $R = 1$ and the center at the origin. Compute the solution of the boundary value problem

$$\Delta u = 0 \text{ in } \Omega, \qquad u(x, y) = x^2 - y^2 - x \text{ on } \partial\Omega.$$

Problem 6.2:
Solve the one-dimensional wave equation $u_{tt} - u_{xx} = g(x, t)$ on $(x, t) \in \mathbb{R} \times \mathbb{R}_+$ for

(a) the homogeneous case $g(x, t) = 0$ and initial conditions $u(x, 0) = \mathbf{e}^x$ and $u_t(x, 0) = \sin(x)$,

(b) $g(x, t) = 0$ and initial conditions $u(x, 0) = \ln(1 + x^2)$ and $u_t(x, 0) = x - 4$,

(c) the inhomogeneous case $g(x, t) = \mathbf{e}^{at}$ and initial conditions $u(x, 0) = u_t(x, 0) = 0$.
Note: For the inhomogeneous wave equation $u_{tt} - u_{xx} = g(x, t)$ with $u(x, 0) = u_t(x, 0) = 0$ the solution is

$$u(x, t) = \frac{1}{2} \int_{s=0}^{t} \int_{y=x-(t-s)}^{x+(t-s)} g(y, s) \, \mathrm{d}y \, \mathrm{d}s.$$

Problem 6.3:
Solve the wave equation in the three-dimensional case with the initial conditions:

(a) $u(x, 0) = 0,\ u_t(x, 0) = x_2$

(b) $u(x, 0) = 0,\ u_t(x, 0) = x_1 x_2$

Problem 6.4:
Compute the solution of the unbounded one-dimensional heat equation

$$u_t - u_{xx} = 0, \quad x \in \mathbb{R},\ t > 0$$

for the initial condition

(a) $u(x, 0) = \mathbf{e}^{3x}$,

(b) $u(x, 0) = 1$ if $x > 0$ and $\varphi(x) = 3$ if $x < 0$.

Problem 6.5:
Find the solution of the following initial-boundary value problem:

$$\begin{aligned} &u_t - u_{xx} = 0, \text{ for } 0 < x < 1,\ t > 0 \\ &u(0, t) = u(1, t) = 0 \\ &u(x, 0) = \sin^2(\pi x) \end{aligned}$$