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## 7. Tutorial on the lecture "Analysis and Numerics of Partial Differential Equations"

Problem 7.1:

Calculate numerical derivatives of

- (a)  $f(x) = e^x$  at x = 0,
- (b)  $f(x) = \ln(x)$  at x = 1,
- (c)  $f(x) = \tan(x)$  at  $x = \frac{\pi}{4}$

using forward, backward and central differences and the stepsize h = 0.1. Compare the results to the exact values.

## Problem 7.2:

Compute an approximation to  $\Delta u$  for  $u(x, y) = \sin(\pi x)\cos(\pi y)$  at (x, y) = (0.4, 0.6) by the 5-point star using  $\Delta x = \Delta y = h = 0.2$ . Compare with the exact value.

## Problem 7.3:

Calculate approximations to  $\sqrt{2}$  by solving for  $x \in [1, 2]$  the Cauchy problem

$$y'(x) = \frac{1}{2y(x)}, \quad y(1) = 1$$

with the following methods:

- (a) explicit Euler,
- (b) implicit Euler,
- (c) Euler-Heun.

Assume a stepsize h = 0.2 and compare the results with  $\sqrt{2}$ .

Problem 7.4:

Solve the boundary value problem

$$-u''(x) + vu'(x) = 1, \quad u(0) = 1, \quad u(1) = 2$$

(a) analytically,

- (b) numerically, assuming the constant  $v \in \{\pm 0.1, \pm 1, \pm 10, \pm 100\}$  and the stepsize h = 0.1. Use the finite difference method,
- (c) try different methods for the first order term.