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8. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 8.1:
Repeat Problem 7.4, but with the boundary condition $u'(1) = 0$ at the right boundary. The rest remains unchanged.

Problem 8.2:
Use the 5-point star and $h_x = h_y = 0.2$ to approximate the solution of

$$\begin{aligned} -\Delta u(x,y) &= 1 && \text{in } \Omega, \\ u(x,y) &= |x| + |y| && \text{on } \partial\Omega \end{aligned}$$

on $\Omega = (0,1)^2$.

Problem 8.3:
Let $u_t + 3u_x = 0$ be given.

(a) Write down an upwind scheme for the PDE using backward differences in x-direction and forward differences in t-direction and discuss this scheme.

(b) Discuss the stability of the numerical solution with respect to the possible stepsizes.

Problem 8.4:
Consider again $u_t + 3u_x = 0$.

(a) Apply the Lax-Friedrich scheme and calculate $U_{2,1}$ to $U_{n-1,1}$, with

$$U_0 = u(0,x) = \max(0, 1 - |x|),$$

for $x \in [0,2]$ and stepsizes $h = 0.4$, $k = 0.1$.

(b) Apply the Lax-Wendroff scheme and calculate $U_{2,1}$ to $U_{n-1,1}$ as in (a).

(c) Compare the results to the analytical solution.