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9. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 9.1:
Let $u_t + xu_x = 0$ be given.

(a) For which interval for x is the applied Lax-Friedrich scheme stable with $k = h = 0.1$?

(b) For $x \in [0, 3]$ and $h = 0.1$ calculate the maximal time stepsize, for which Lax-Friedrich is still stable.

(c) Would the Lax-Wendroff scheme converge for this problem?

Problem 9.2:
Consider the boundary value problem

$$\begin{aligned} u_{xx} + 3u_{xy} - 7u_{yy} - u_y - u &= 3 && \text{in } \Omega, \\ u(x, y) &= g(x, y) && \text{on } \partial\Omega \end{aligned}$$

(a) Compute the difference stencil for the inner points with step sizes h_x and h_y.

(b) Give the system of linear equations for $\Omega = (0, 1)^2$, $h_x = h_y = 0.25$ and $g(x, y) = |x - y|$.

Problem 9.3:
Let the domain $\Omega = (0, 2) \times (0, 1) \cup (0, 1) \times [1, 2)$ and the boundary value problem

$$\begin{aligned} u_{xx} - u_{yy} + 4u_y - u_x &= 1 && \text{in } \Omega, \\ u(x, y) &= 5 && \text{on } \partial\Omega \end{aligned}$$

be given. Assume a uniform grid with step size $h_x = h_y = 0.2$.

(a) Sketch the domain and the grid and number the grid nodes.

(b) Write down the difference stencil.

(c) Determine the discretization matrix and the corresponding right-hand side.

(d) Compute and plot an approximation to the solution.

Problem 9.4:
Solve the boundary value problem

$$\begin{aligned} -u_{xx}(x, y) + 3u_{yy}(x, y) &= 1 && \text{in } \Omega, \\ u(x, y) &= 2|x| + y && \text{on } \partial\Omega \end{aligned}$$

with $\Omega = (-1, 1) \times (-1, 1)$ and $h_x = h_y = 0.2$ for a uniform grid.