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10. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 10.1:
Determine whether the following matrices are irreducible or not.

(a) $A = \begin{pmatrix} 1 & 7 & 3 & 0 \\ 0 & -1 & 2 & 0 \\ 9 & 0 & 0 & 5 \\ 0 & 3 & 0 & 8 \end{pmatrix}$ (b) $B = \begin{pmatrix} 3 & -3 & 5 & 9 \\ 0 & 1 & 0 & -2 \\ 9 & 3 & 7 & 2 \\ 0 & 7 & 0 & 6 \end{pmatrix}$

(c) The discretization matrix of $\Delta u = 0$ for $\Omega = (0,1)^2$ with $h = 0.2$.

Problem 10.2:
Let be given the following parabolic PDE with initial and boundary conditions:

$$\begin{aligned} &u_t - u_{xx} = 3 \text{ in } (0,1) \times (0,20), \\ &u(x,0) = \begin{cases} 2x, & x \in (0, 0.5], \\ 2(1-x), & x \in (0.5, 1), \end{cases} \\ &u(0,t) = u(1,t) = 0. \end{aligned}$$

Calculate the first time layer with $h = 0.25$ and $k = 0.06$.

(a) Use the explicit Euler-method
(b) Use the Crank-Nicolson-method.
(c) Use the implicit Euler-method.

Problem 10.3:

(a) Are the solutions of Problem 11.2 stable?
(b) What are the conditions that the solutions are stable for $h = 0.1$?

Problem 10.4:
Let the following initial boundary value problem be given

$$\begin{aligned} &u_t - u_{xx} = 5 \text{ in } (-1,1) \times (0,10), \\ &u(x,0) = 10x^2, \text{ for } x \in (-1,1), \\ &u(-1,t) = u(1,t) = 10. \end{aligned}$$

Calculate the first time layer with $h = 0.25$ and $k = 0.01$.