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11. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 11.1:
Let the following parabolic initial-boundary value problem be given

$$\begin{aligned} u_t(x,y,t) &= 2\Delta u(x,y,t) && \text{for } (x,y) \in \Omega = (-1,1)^2,\ t \in (0,10),\\ u(x,y,0) &= \max\big(0,(1-x^2)(1-y^2)\big) && \text{for } (x,y) \in \Omega,\\ u(x,y,t) &= 0 && \text{for } (x,y) \in \partial\Omega,\ t \in (0,10). \end{aligned}$$

Use the explicit Euler-method and set up the discretization matrix with $h_x = h_y = 0.25$ and $k = 0.03$. Calculate the approximations for the first time layer.

Problem 11.2:
Solve the following hyperbolic initial-boundary value problem

$$\begin{aligned} u_{tt}(x,t) &= 4u_{xx}(x,t) && \text{for } x \in \Omega = (0,1),\ t \in (0,10),\\ u(x,0) &= \exp\big(-40(x-0.5)^2\big) && \text{for } x \in \Omega,\\ u_t(x,0) &= 0 && \text{for } x \in \Omega,\\ u(0,t) &= u(1,t) = 0 \text{ for } x \in \partial\Omega,\ t \geq 0. \end{aligned}$$

Use finite differences with $h_x = 0.2$ and $k = 0.05$. Compare your approximation with the analytical solution.

Problem 11.3:
What can be said about the numerical stability of problem 12.2? What is the maximal stepsize in time direction regarding stability?

Problem 11.4:
Implement a code to approximate the solution of the initial-boundary value problem

$$\begin{aligned} u_{tt} - \Delta u &= 0 && \text{for } x \in \Omega = (-1,1)^2 \quad ,\\ u(x,y,0) &= \exp\big(-20(x^2+y^2)\big) && \text{for } x \in \Omega,\\ u_t(x,y,0) &= 0 && \text{for } x \in \Omega,\\ u(t,x) &= 0 && \text{for } x \in \partial\Omega,\ t > 0. \end{aligned}$$

Use a grid with $N = 31$ nodes in each direction. Compute approximations for $t \in [0,5]$ with $k = 0.05$. Use a number of 100 eigen modes to approximate the initial displacement.