University of Rostock
Institute of Mathematics
PD Dr. M. Sawall
M.Sc. T. Andersons

Rostock, 10.07.2025

12. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 12.1:

Let the following parabolic initial-boundary value problem be given

$$\begin{aligned} &u_t - u_{xx} = 5 \text{ in } (-1,1)\times(0,10) \\ &u(x,0) = 10x^2, \text{ for } x \in (-1,1) \\ &u(-1,t) = u(1,t) = 10. \end{aligned}$$

Use the method of lines with stepsizes $h = 0.5$ and $k = 0.01$. To this end

(a) transform the PDE into an ODE system and

(b) use Heun's method to find the solution for the first layer.

Problem 12.2:

Transform the hyperbolic initial-boundary value PDE

$$\begin{aligned} &u_{tt} = 4u_{xx}, && \text{for } x \in \Omega = (0,1),\ t > 0 \\ &u(x,0) = \exp(-40(x-0.5)^2) && \text{for } x \in \bar{\Omega} \\ &u_t(x,0) = 0 && \text{for } x \in \bar{\Omega} \\ &u(0,t) = u(1,t) = 0 && \text{for } x \in \partial\Omega,\ t \geq 0 \end{aligned}$$

into an ODE system.

Problem 12.3:

Give a weak formulation of $-\big((x^2+1)u(x)'\big)' + u(x) = x$ for $\Omega = (0,1)$ with $u(0) = u(1) = 0$.

Problem 12.4:

Show that the boundary value problem (clamped beam)

$$u^{(4)}(x) = f(x) \text{ in } \Omega, \qquad u(x) = u'(x) = 0 \text{ on } \partial\Omega$$

with $\Omega = (0,1)$ has the weak formulation

$$\int_0^1 u''(x)v''(x) = \int_0^1 f(x)v(x) \qquad \forall v \in \big\{v \in \mathcal{C}^2(\Omega) \cap \mathcal{C}(\bar{\Omega}) :\ v(0) = v(1) = v'(0) = v'(1) = 0\big\}.$$