4. Additional tasks for exercise on "Introduction to Numerical Mathematics"

Problem 15:

Apply Jacobi's method and Gauss-Seidel's method to solve Ax = b with

$$A = \begin{pmatrix} -4 & 1 & 0 & 1 \\ 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 1 & 0 & 0 & 4 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

Use $x^{(0)} = (1, 1, 1, 1)^T$ as starting vector and compute 2 steps for each method.

Problem 16:

Does the iterations in Problem 15 converge for each starting vector?

Problem 17:

Apply an a-priori estimation of the error in order to determine how many steps are necessary to reach an approximation with $||x^* - x^{(k)}||_2 < 10^{-5}$ in Problem 15.