University of Rostock
Institute of Mathematics
PD Dr. M. Sawall

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8. Additional tasks for exercise on „Introduction to Numerical Mathematics“

Problem 31:

(a) Apply one step of Newton's method for $x^{(0)} = (0.98, 0.32)^T$ to solve $f(x) = 0$ for (Kantorowitsch, Akilow)

$$f(x) = \begin{pmatrix} 3x_1^2 x_2 + x_2^3 - 1 \\ x_1^4 + x_1 x_2^3 - 1 \end{pmatrix}.$$

(b) Using a computer apply 4 stepa of Newton's method for (Brown, Conte)

$$g : \mathbb{R}^3 \to \mathbb{R}^3, \quad g(x) = \begin{pmatrix} 3x_1 + x_2 + 2x_3^2 - 3 \\ -3x_1 + 5x_2^2 + 2x_1 x_3 - 1 \\ 25x_1 x_2 + 20x_3 + 12 \end{pmatrix}$$

using $x^{(0)} = (0, 0, 0)^T$.

Problem 32:

Using the method of steepest descend without stepsize control compute two steps to minimize

$$F : \mathbb{R}^2 \to \mathbb{R}, \quad F(x) = \|f(x)\|_2^2$$

using $f(x)$ from the previous problem and $x^{(0)} = (0.98, 0.32)^T$.

Problem 33:

Generate a fractal based on the solutions of the equation $z^3 - 2 = 0$ for $z \in \mathbb{C}$. The decomposition of $z = x + \mathbf{i}y$ as well as $z^3 - 2$ into its real and imaginary parts results in

$$0 = (x + \mathbf{i}y)^3 - 2 = (x^3 - 3xy^2 - 2) + \mathbf{i}(3x^2 y - y^3).$$

To generate the fractal, compute the solutions of

$$f(x, y) = \begin{pmatrix} x^3 - 3xy^2 - 2 \\ 3x^2 y - y^3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Use Newton's method. Define a grid with $h = 0.01$ for $[-1.5, 1.5]^2$ and use the single grid points as starting vectors for Newton's method. Now graphically represent the catchment areas of the different solutions by coloring the catchment areas.