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2. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 2.1:
Let f and g be arbitrary differentiable functions. Find the second order partial differential equation which is independet of f and g and has the general solution

(a) $u(x, y) = f(x) + g(y)$,

(b) $u(x, y) = f(x) \cdot g(y)$.

Problem 2.2:
Consider the 2D vector field $\overrightarrow{v} = \big((x+1)y,\ x(y+1)\big)^T$.

(a) Compute the integral curves of $\overrightarrow{v}$.

(b) Determinde the general solution $z(x, y)$ of $(x+1)yz_x + x(y+1)z_y = 0$.

Problem 2.3:
Solve the PDE

$$3yu_x - 2xu_y = 0$$

and find the particular solutions that satisfy the initial condition

(a) $u(x, y) = x^2$ on the line $y = x$ resp.

(b) $u(x, y) = 1 - x^2$ on the line $y = -x$.

Problem 2.4:
Compute the integral curves of the following vector fields and find the curves that pass through the point $P = (1, 1, 1)$

$$\text{(a)}\ \overrightarrow{v} = \begin{pmatrix} z \\ y \\ x \end{pmatrix}, \qquad \text{(b)}\ \overrightarrow{v} = \begin{pmatrix} x \\ -y \\ y^2(1-z) \end{pmatrix}, \qquad \text{(c)}\ \overrightarrow{v} = \begin{pmatrix} y+z \\ z+x \\ x+y \end{pmatrix}.$$