

3. Tutorial on the lecture „Analysis and Numerics of Partial Differential Equations“

Problem 3.1:

(a) Find the integral surfaces of $\vec{v} = (x^2, y^2, (x+y)z)^T$.

(b) Calculate the surface containing

$$(i) \ x = y, \ z = 1, \quad (ii) \ x = y = z, \quad (iii) \ x = 1, \ y = z.$$

Problem 3.2:

Calculate the integral curves with either the integral surfaces **or** using a system of ODEs to get the parametric form.

$$(a) \ \vec{v} = \begin{pmatrix} x \\ x+y \\ x+y+z \end{pmatrix}, \quad (b) \ \vec{v} = \begin{pmatrix} x^2 - 1 \\ (y^2 + 1)(x + 1) \\ xz + x - z - 1 \end{pmatrix}.$$

Problem 3.3:

(a) Find the integral surfaces of the vector field

$$\vec{v} = \begin{pmatrix} (x+1)y \\ y^2 + 1 \\ y(z-1) \end{pmatrix}.$$

(b) Compute the solution $z = z(x, y)$ of the inhomogeneous PDE

$$(x+1)yz_x + (y-1)(y+1)z_y = y(z-1).$$

(c) Further, solve the PDE

$$(x+1)yu_x + (y^2+1)u_y + y(z-1)u_z = 0$$

for $u = u(x, y, z)$.

Problem 3.4:

Find the solution $z = z(x, y)$ for the PDE

$$y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = -1, \quad 0 < y < x$$

with the initial condition $z(x, 0) = \ln x$ for $x > 0$.