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Suggested solutions for self-study & additional practice for the 5. Tutorial

Sample solution for the additional exercise 19:
We are locking for a function $p(x) = a + bx$ resp. $q(x) = a + bx + cx^2$. The measures are

x	0	1	2	3	4	5
y	225	230	230	200	160	110

For a quadratic function:

$$\begin{aligned}
&y(x) = a_0 + a_1 x + a_2 x^2, \\
&(0, 225): \qquad 225 = a_0 + 0 \cdot a_1 + 0^2 \cdot a_2 \\
&(0, 230): \qquad 230 = a_0 + 1 \cdot a_1 + 1^2 \cdot a_2 \\
&(0, 230): \qquad 230 = a_0 + 2 \cdot a_1 + 2^2 \cdot a_2 \\
&\vdots \\
&(0, 110): \qquad 110 = a_0 + 5 \cdot a_1 + 5^2 \cdot a_2
\end{aligned}$$

Linear least-squares problem $Ax = b$ with

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{pmatrix}, \qquad b = \begin{pmatrix} 225 \\ 230 \\ 230 \\ 200 \\ 160 \\ 110 \end{pmatrix}.$$

Equations:

$$A^T A = \begin{pmatrix} 6 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{pmatrix} A^T b = \begin{pmatrix} 1056 \\ 1985 \\ 5785 \end{pmatrix}$$

$$A^T A \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = A^T b.$$

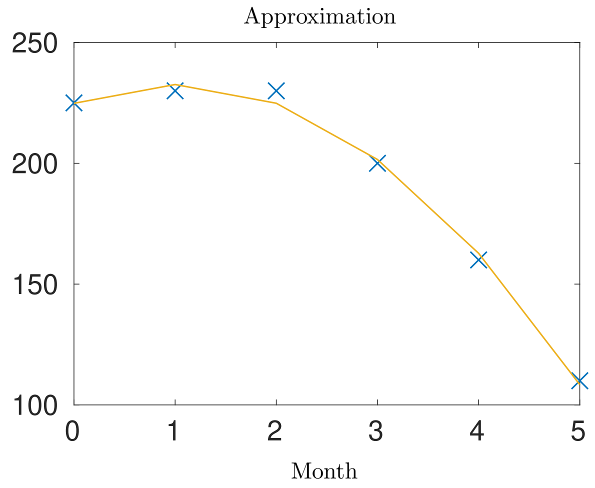
Solution:

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 214.21 \\ 45.61 \\ -16.61 \end{pmatrix}.$$

Polynomial of degree 2:

$$y(x) = 214.21 + 45.61x - 16.61x^2.$$

A graphic for the quadratic function:



Sample solution for the additional exercise 20:
Gershgorin's disks are

$$
\begin{aligned}
&A: \; K_1 = K_3 = \big\{z: \; |z+4| \leq 1\big\}, \quad K_2 = \big\{z: \; |z+4| \leq 2\big\},\\
&B: \; K_1 = \big\{z: \; |z-1| \leq 5\big\}, \quad K_2 = \{z: \; |z-5| \leq 10\}, \; K_3 = \{z: \; |z-9| \leq 15\},\\
&C: \; K_1 = \{z: \; |z-1| \leq 1.2\}, \; K_2 = \{z: \; |z+1| \leq 0.1\}.
\end{aligned}
$$

As for A holds $0 \notin \cup K_i$ the matrix A is invertible for sure, for B and C this is not sure as $0 \in \cup K_i$ in both cases. Estimations for the eigenvalues of A^{-1} can be computed as follows: A is symmetric and all eigenvalues are real, thus we can use inverses of the extremal elements of K_i and get

$$\tilde{K}_1 = \left[\frac{1}{9}, \frac{1}{3}\right], \quad \tilde{K}_2 = \left[\frac{1}{5}, 1\right], \quad \tilde{K}_3 = \left[-1, -\frac{1}{7}\right],$$