University of Rostock
Institute of Mathematics
PD Dr. M. Sawall

Rostock, 18.10.2023

1. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 1:
Check for which values $n \leq 6$ the calculation of

$$S_n = \sum_{i=1}^{n} \frac{1}{n}$$

will give the correct result $S_n = 1$. Note the floating point arithmetic $x \oplus y = \mathrm{rd}(x + y)$, that is, floating point numbers are first added exactly and then the rounding operator is applied.

Problem 2:
Sum up at least ten billion terms of the harmonic series. Compare the results for summation in forward and backward direction and explain the difference.

Problem 3:
Calculate the value of the polynomial

$$y(x) = 1.0837x^4 + 2.7911x^3 + 0.75149x^2 - 5.8205x - 7.6123$$

for $x = 1.4935$ using the following two algorithms:

Algorithm 1	Algorithm 2
$y_1 = x$	$y_1 = 1.0837x$
$y_2 = xy_1$	$y_2 = (y_1 + 2.7911)x$
$y_3 = xy_2$	$y_3 = (y_2 + 0.75149)x$
$y_4 = xy_3$	$y_4 = (y_3 - 5.8205)x$
$y = 1.0837y_4 + 2.7911y_3 + 0.75149y_2 - 5.8205y_1 - 7.6123$	$y = y_4 - 7.6123$

Compare both algorithms according their computation expense and memory consumption.

Problem 4:

a) Show that the inequality

$$\frac{1}{1+2x} - \frac{1-x}{1+x} > 0$$

holds for each positive real number, $x \in \mathbb{R}$, $x > 0$. Calculate the left-hand side of the inequality for $x = 10^{-10}$ and provide a numerically stable form of the inequality.

b) Show that the function

$$f(x) := x(\exp(x^{-1}) - 1)$$

has the limit 1 for $x \to \infty$. Use a calculator to evaluate the function $f(x)$ for $x = 10^j$ ($j = 5, \ldots, 15$). Provide a numerically stable form using a series expansion.

Problem 5:
Let the number $x > 0$ be assigned a relative measurement error of at most 10%. How do the relative errors for

(a) $f(x) = \frac{1}{x}$, (b) $f(x) = \ln(x)$?

At which points x does a particularly large gain occur in (b)?