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2. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 6:
At $x = \sqrt{2}$, consider the three functions

$$f_1(x) = (x-1)^6, \qquad f_2(x) = 99 - 70x, \qquad f_3(x) = \frac{1}{99+70x}.$$

Show analytically that

$$f_1(\sqrt{2}) = f_2(\sqrt{2}) = f_3(\sqrt{2}).$$

Calculate the absolute and relative condition numbers for each of the expressions.

Problem 7:
Consider the system of linear equations $Ax = b$ with

$$A = \begin{pmatrix} \alpha & 1 & 0 \\ 1 & 0 & 1 \\ 2 & \alpha & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$$

for a parameter $\alpha \in \mathbb{R}$.

(a) Calculate the solution x depending on the parameter α.
(b) For which values α does no solution exists?
(c) Which values α cause a Gaussian elimination that requires pivoting?
(d) Determine a right-hand-side $b \neq (0,0,0)^T$ such that there exists a solution for any choice of the parameter α.

Problem 8:
Write a program using Gaussian elimination (with pivotization) to solve systems of linear equations. Test your implementation of the Gaussian algorithm on systems of the form $Ax = b$ with

$$A_{ij} = (1 + |i-j|)^{-1}, \quad b_i = 1, \quad i,j = 1 \dots n.$$

Count the numbers of flops and plot them against n for $n = 1, 2, 3, \dots 12$!

Problem 9:
Consider an electric circle with 5 resistors placed on the edges of the square $ABCD$ and on its diagonal BD. Resistor $R_1 = 3\Omega$ connects A with B, resistor $R_2 = 8\Omega$ connects B with C, resistor $R_3 = 3\Omega$ connects C with D, resistor $R_4 = 8\Omega$ connects D with A and resistor $R_5 = 3\Omega$ connects B with D. A current of 1A enters the electric circle at A and leaves it at C.

Use Kirchhoff's laws to derive a system of linear equations to calculate the five currents passsing the resistors and compute the solution using the program from task 8.