University of Rostock
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3. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 10:
Determine the LU-decomposition of the following matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \\ 1 & 16 & 81 & 256 \end{pmatrix}$$

Solve the linear system $Ax = b$ for $b = (3, 1, -15, -107)^T$ as well as for $b = (10, 20, 46, 116)^T$.

Problem 11:
Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}.$$

Calculate $\text{cond}_1(A)$, $\text{cond}_2(A)$ and $\text{cond}_\infty(A)$.

Problem 12:
Which value of c leads to zero in the second pivot position of the matrix:

$$A = \begin{pmatrix} 1 & c & 0 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{pmatrix}$$

For exactly that value of c, calculate a PLU decomposition such that $PA = LU$!

Problem 13:
Consider a band-matrix with with α lower subdiagonals and β upper subdiagonals for $\alpha, \beta \in \mathbb{N}$. The LU-decomposition return two band-matrices. How many subdiagonals do not vanish if the Gaussian-elimination is done

(a) without pivot search and

(b) with pivot search?

Problem 14:
Are the following symmetric matrices positive/negative definite?

$$H_1 = \begin{pmatrix} 4 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}, \qquad H_2 = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 17 \end{pmatrix}.$$