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5. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 19:
Peter measured the braking distances of his bike at different speeds and got the following values

v [m/s]	3	6	8	14
s [m]	0.1	2.5	5	12

He wants to approximate them by a polynom of degree less or equal two in terms of a least-squares fit.

(a) Set up the overdetermined system of linear equations.
(b) Derive the linear system of equations for the necessary condition for the unkown parameter.
(c) What is the approximate value to expect for $v = 11$?

Problem 20:
Compute the QR-decomposition of the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 1 & 6 \\ 1 & 8 \end{pmatrix}^T$$

using Householder transformations. Compute the least-squares solution of $Ax = b$ for $b = (0.1,\ 2.5,\ 5)^T$.

Problem 21:
Using Gershgorin's circle theorem, calculate intervals for the eigenvalues of

$$A = \begin{pmatrix} 6 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & -4 \end{pmatrix}, \qquad B = 16 \begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & -1 \\ 0.1 & 2 \end{pmatrix}.$$

So which of the matrices are safely invertible without further calculations? Estimate for these the eigenvalues of the inverses.

Problem 22:
Consider the matrix $A = h^{-2}\mathrm{tridiag}(-1, 2, -1) \in \mathbb{R}^{n\times n}$ for $n \in \mathbb{N}$ and $h = 1/(n+1)$. Show that the columns of the matrix $X \in \mathbb{R}^{n\times n}$ with the elements

$$X_{jk} = \sin(k\pi jh), \quad j, k \in \{1, \dots, n\},$$

are the eigenvectors of A and compute the eigenvalues.

Problem 23:
Consider a system of n equal masses with $m = M$ in a row. Each mass is connected to its left and right neighbors by equal springs. The first and last masses are connected to the left and right walls.

Let $z \in \mathbb{R}^n$ be the vector of deflections from the equilibrium positions. Find the equations of motion describing the mechanical system and state them in the form $m\ddot{z} = Az$.