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6. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 24:
Apply the power method as well as the inverse power method to

$$A = \begin{pmatrix} 6 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & -4 \end{pmatrix}.$$

Calculate three steps starting from $x^{(0)} = \sqrt{1/3} \cdot (1,1,1)^T$ for the power method as well as one step starting from $z^{(0)} = \sqrt{1/2} \cdot (1,-1,0)^T$ for the inverse vector iteration. Give the approximations for the largest and smallest eigenvalues of A as well as the eigenvectors. Compare the approximations to the values

$$\lambda_1 = -4.4708, \quad \lambda_2 = 2.7149, \quad \lambda_3 = 6.7559.$$

Problem 25:
Apply 3 steps of the shifted inverse power method for $\mu = -4$ to A from task 24. Use the starting vector $x^{(0)} = \sqrt{1/3} \cdot (1,1,1)^T$ and the LU-decomposition of $A - \mu I$ with

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \qquad U = \begin{pmatrix} 2 & 1 & 2 \\ 0 & -1.5 & 0 \\ 0 & 0 & -10 \end{pmatrix}.$$

Problem 26:
Consider once again the matrix A from task 24. Use a computer and

(a) apply 3 steps of the QR-algorithm,

(b) apply 3 steps of Jacobi's iteration.

Problem 27:

(a) Apply the QR-algorithm to find all eigenvalues of the matrix $E = (E_{ij}) \in \mathbb{R}^{6x6}$ defined by

$$E_{ij} = \begin{cases} 8 - i & \text{if } i = j, \\ -1 & \text{if } |i-j| = 1, \\ 0 & \text{otherwise.} \end{cases}, \qquad i,j = 1,\dots,6.$$

Determine the eigenvectors as well!

(b) Apply Jacobi's method to compute the eigenvalues and eigenvectors of the matrix

$$A = (n+1)^2 \text{tridiag}(1,-2,1) \in \mathbb{R}^{n \times n}$$

for $n = 30$. Plot the eigenvectors associated to λ_i, $i \in \{1,2,29,30\}$, against

$$x \in \mathbb{R}^n, \qquad x_i = \frac{i}{(n+1)^2}.$$