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Rostock, 06.12.2023

8. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 32:
Let $\varphi : \mathbb{R} \to \mathbb{R}$ be continuously differentiable with the fixed point $\bar{x}$ and $|\varphi'(\bar{x})| \neq 1$. Consider the fixed-point forms

$$\begin{array}{ll} 1. & x_{k+1} := \varphi(x_k) \\ 2. & x_{k+1} := \varphi^{-1}(x_k) \end{array} \qquad k = 0, 1, 2, \ldots.$$

Show that at least one of the fixed-point forms converge to the fixed point $\bar{x}$.

Problem 33:
Formulate a fixed-point iteration to determine the point of intersection (x^*, y^*) defined by the two equations

$$f(x) = 2 \cdot \exp(-x), \qquad g(x) = \sqrt{1+x}$$

Show that the assumptions of the Banach fixed-point theorem are fulfilled and thus it holds

$$x^* = \lim_{k\to\infty} x_k \,.$$

Problem 34:
Calculate 10 steps using the simplified Newton method for problem 31 with $x_0 = 2$. Evaluate f' in the first and in the sixth iteration. Compare the results with those of problem 31.

Problem 35:
Compute approximation to the two zeros of

$$f : \mathbb{R}^2 \to \mathbb{R}^2, \qquad f(x,y) = \begin{pmatrix} \exp(x) - y \\ y^2 - x - 3 \end{pmatrix}.$$

(a) Give the multidimensional iteration prescription of Newton's method for this $f(x, y)$.

(b) Apply one step of Newton's method for $x^{(0)} = (-1, 0)^T$.

(c) Further, apply also one step of Newton's method for $x^{(0)} = (1, 4)^T$.

(d) For which $(x, y) \in \mathbb{R}^2$ is $f'(x, y)$ not invertible?

(e) Use an (unfavorable) alternative and apply (compute two steps to for $x^{(0)} = (-1, 0)^T$) the method of steepest descend without stepsize control to minimize

$$F : \mathbb{R}^2 \to \mathbb{R}, \quad F(x) = \frac{1}{2}\|f(x)\|_2^2.$$

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.