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10. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 40:
Calculate the Newton interpolation polynomial for

$$p(0)=1,\ p(-1)=3,\ p(1)=15,\ p'(-1)=-12,\ p'(1)=40.$$

Compute $p'(0)$.

Problem 41:
For a function $u:\mathbb{R}^2\to\mathbb{R}$ the following points are known

$$u(0,1)=3,\ u(1,1)=6,\ u(0,2)=7,\ u(1,2)=11,\ u(1.5,1.5)=10.75.$$

Further u should satisfy $\Delta u(1.5,1.5)=0$.

Set up the 6 equations necessary to determine the two-dimensional polynomial of degree less than or equal to two using this information but do not finally compute the polynomial.

Problem 42:

(a) For $f(x)=\tan(x)$, determine approximations to $f'(x_0)$ for $x_0=0.125\pi$ using forward, backward and central difference and for $f''(x_0)$ using second order central difference. Compare the approximate values with the actual ones. Use $h=10^{-3}$ for all approximations.

(b) Determine for $u(x,y)=\sin(x)\exp(-y^2)$ approximations to $u_{xx}(x_0,y_0)$, $u_{xy}(x_0,y_0)$ and $u_{yy}(x_0,y_0)$ for $(x_0,y_0)=(1.25,0.75)$. Use $h_x=h_y=h$ for $h=10^{-3}$. What are the absolute errors.

(c) Compute an approximation of the Jacobian $J_f(-1,0)$ for f from problem 31 at the point $x=(-1,0)$. Use forward differences and $h=10^{-4}$. Compare the approximation with the exact value and calculate $\mathrm{cond}_\infty(A)$.

Problem 43:
When modeling the deformation of a one-dimensional beam, a difference quotient is needed for approximation $u^{(4)}(x)=\frac{\mathrm{d}^4u}{\mathrm{d}x^4}(x)$. Derive this using Taylor expansions of $u(x\pm h)$ and $u(x\pm 2h)$ and $u(x)$. What is the error order of the approximation.

Use the difference quotient to approximate $u^{(4)}(0.25)$ for $u(x)=-\cos(\pi x)$ with $h=10^{-k}$, $k=1,\dots,4$. Give the absolute errors.

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.