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11. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 44:
For $f(x) = \sin(\pi x)$ use the composite trapezion rule as well as the composite Simpson's rule to approximate

$$I = \int_0^1 f(x)\,\mathrm{d}x.$$

(a) Apply both rules for $n = 6$ and give the errors $|I - T_6|$ and $|I - S_6|$.

(b) Determine n such that the error of the composite trapezion rule resp. the composite Simpson's rule is smaller than 10^{-6}.

Problem 45:
Compute appoximations to

$$\int_0^1 \sin(\pi x)\,\mathrm{d}x$$

(a) using Gaussian quadratur rules with 2 nodes,

(b) using Gaussian quadratur rules with 3 nodes,

(c) using composite Gaussian quadratur rules with 2 nodes and 3 intervals.

Problem 46:
The height of a barrel is 0.6 m and its diameter follows the function $d(h) = 0.3 - 0.4(h - 0.6)^2$. Determine the volume of this barrel. To this end calculate the volume of the solid of revolution that results when the function $d(h)$ rotates around the h-axis. Use composite Simpson's rule with $n = 4$ intervals.

Problem 47:
Compute approximations to

$$I = \frac{1}{\ln(2)} \int_0^{\pi/2} \frac{x}{1 + \sin(x)}\,\mathrm{d}x$$

using Romberg's scheme. Apply trapezion rule as basis method with the starting stepsize $h_0 = \pi/2$ as well as the final stepsize $\pi/8$.

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.