University of Rostock
Institute of Mathematics
PD Dr. M. Sawall

Rostock, 10.01.2024

12. Tutorial on the lecture „Introduction to Numerical Mathematics“

Problem 48:
Compute an approximation to $\int_{-\infty}^{\infty} \exp(-x^2)\,\mathrm{d}x$. To this end use the coordinate transformation $x(z) = \tan(z)$ with

$$x' = \frac{\mathrm{d}x}{\mathrm{d}z} = \frac{1}{\cos(z)^2} \quad \Rightarrow \quad \mathrm{d}x = \frac{1}{\cos(x)^2}\,\mathrm{d}z$$

and Simpson's rule for $n = 6$. Give the errors.

Problem 49:
Consider the initial value problem

$$y'(x) = -\sin(x)y(x) \text{ for } x \in [0,1], \quad y(0) = 1.$$

(a) Compute three steps of Euler's method with $h = 0.1$.

(b) Compute two steps of Heun's method with $h = 0.15$.

(c) Apply the 4-stage Runge-Kutta method RK-4 and compute one step with $h = 0.3$.

Problem 50:

```
1  n = 51;
2  h = 10/(n-1);
3  X = 0:h:10;
4  Y(1) = exp(cos(0));
5  for i=1:n-1
6      K1 = -sin(X(i))*Y(i);
7      K2 = -sin(X(i)+h/2)*(Y(i)+h/2*K1);
8      K3 = -sin(X(i)+h/2)*(Y(i)+h/2*K2);
9      K4 = -sin(X(i)+h)*(Y(i)+h*K3);
10     Y(i+1) = Y(i)+h/6*(K1+2*K2+2*K3+K4);
11 end
```

Refer to the relevant line(s) for each of the following questions.

(a) Which initial value problem is solved here?

(b) What is the numerical method used and what is the step size?

(c) For the selected step size, the maximum error is $7.1 \cdot 10^{-6}$. Which error is to be expected due to the consistency order of the method, if $n = 101$ would be chosen?

Problem 51:
Consider the initial value problem $y'(x) = \lambda y(x)$ for $x \in [0,1]$ and $y(0) = 1$.

(a) Apply Euler's method with $h \in \{1, 10, 100\}$.

(b) Apply Heun's method with $h \in \{1, 0.1, 0.01\}$.

(c) Apply the implicit Euler method with $h \in \{1, 0.1, 0.01\}$.

Compare the results with the analytical value.

The tasks are intended both for processing in the seminars and for independent practice. Especially the 90 minutes of an exercise are sometimes not sufficient to discuss and work on all tasks.